

Influence of a Uniform Horizontal Magnetic Field in a Square Cavity with Heat Sources and Hybrid Nanofluid

Belkacem Ait taleb¹, Adel Sahi², Djamel Sadaoui³

^{1,2,3} Université de Bejaia, Faculté de Technologie, Laboratoire de Mécanique Matériaux et Energétique, Faculté de Technologie, Université de Bejaia, 06000 Bejaia, Algérie.

E-mail: belkacem.aittaleb@univ-bejaia.dz, adel.sahi@univ-bejaia.dz, djamel.sadaoui@univ-bejaia.dz

Abstract: Natural convection is studied in a square cavity filled with a hybrid nanofluid in the presence of a horizontal magnetic field. Cooling temperatures are applied to the left and right sides of the cavity, while the other walls are adiabatic. Inside the cavity, four uniformly distributed square blocks generate heat flux. The finite volume method, using the SIMPLE algorithm and considering the Boussinesq approximation, is employed for the analysis. The results are presented in terms of streamlines, isotherms and profiles of the Nusselt number, Hartmann number, Rayleigh number and volume fraction. The results indicate that an increase in Rayleigh number and volume fraction improves heat transfer within the cavity. Conversely, the application of the magnetic field reduces heat transfer efficiency.

Keywords: MHD; Heat source; Square cavity; Natural convection; Hybrid Nanofluid.

1. INTRODUCTION

The Heat transfer is an important role in fluid mechanics and industrial fields (Ali et al., 2016)(Khodabandeh et al., 2018). The convection heat transfers is the action of buoyancy force caused by thermal gradient, this process provided in many applications like cooling of solar technologies (Kavusi & Toghraie, 2017), heat exchangers (Marandi et al., 2018) and so other. Choi (Choi, 1995) develop a fluid combined with nanoparticles which is named by nanofluid that improves the characteristics of the base fluid.

Several studies focused in general on sensitive parameters, either to highlight the development of an effect, or to improve a specific characteristic. Rowsanara Akhter and al (Akhter et al., 2023) studied a mixed convective in square cavity rumpled by a hybrid-nanofluid with two rotating rough cylinders in presence of an external magnetic field using finite element method. They found that speed rotation convective flux accelerates with cylinder rotation speed and decreases with increasing magnetic field strength and volume fraction. T. Armaghani and al (Chamkha et al., 2021) study of mixed conduction in the presence of a magnetic field and a hybrid Al₂O₃-Cu/Water nanofluid inside an L-shaped cavity with two uniform heat sources solved by the finite-difference method. The results show that the best heat transfer performance results in the maximum amount of sink power. N. Vishnu Ganesh (Ganesh et al., 2020) a numerical study on a square cavity composed of different obstacles filled with Al₂O₃-H₂O nanofluid heated at the bottom and cooled by vertical walls uses Galerkin's finite element method. The results show that Nusselt numbers increase with the volume fraction of nanoparticles and that a higher heat transfer is obtained in the case of triangular obstacles. Sadia Tasnim and al (Tasnim et al., 2023) presented the natural convective of nanofluid TiO₂-water confined in a tilted square cavity with

multiple heat-generating and a constant magnetic field is solved by the finite element . have been observed that position of the heat-generating elements and the inclination of cavity influence the thermal performance. M. A. Mansour and al (Ismael et al., 2016)(Mansour et al., 2016) presented mixed convection of Cu-water nanofluid inside of square cavity with moving of the top and bottom walls with heat source in part of the left wall and internal heat generation or absorption using the finite volume method. has been find that increase in the heat source length cased a decrease in the shear-driven force and local Nu number. Rumman Hossain and al (Hossain et al., 2022) have been used Oil-based nanofluids in a square cavity with semicircular heater in the bottom with using a magnetic field, the method of resolution based on The Galerkin finite elements. the thermophysical properties enhances in the nanofluid than the base fluid. Fatih Selimefendigil, and Hakan F. Öztöp (Selimefendigil & Öztöp, 2019) has been used nanofluid in an inclined L-shaped cavity with an region elastic wall to study the mixed convective and effect of magnetic field and heat generation by finite element technique with the Arbitrary-Lagrangian– Eulerian method. Elastic walls have a significant effect on convective heat transfer at the lowest values of Richardson and modulus of elasticity.

According to the literature many researchers investigate for the process of heat transfer. The present study evaluates the systems of natural convection in square cavity with two cold wall left and right with isothermal top and bottom wall, inside are rumpled by hybridnanofluid Cu/Al₂O₃/H₂O at different fraction in presence of magnetic field in left side, with forth uniform element of heat generation, used the finite volume method and Boussinesq approximation to observe the manifestation of fluid flow and the effect of magnetic field, volume fraction of hybridnanofluid on the heat transfer in presence of heat generation.

2. PROBLEM FORMULATION

The modeling was carried out on a 2D Cartesian coordinate system with laminar and incompressible flow as depicted in figure 1. natural convection, is the physical phenomenon how create the flow of hybrid nanofluid (Al₂O₃-Cu/H₂O) in cavity. The wall subdivise at cold constant temperature T_c (left and right) and adiabatic side (top and bottom). A uniform horizontal magnetic field is applied with a constant magnitude B₀. Four square block elements, having the same shape, material, and constant heat generation flux, are deposited at equal distances from the edges of the cavity. Using the Boussinesq approximation to validate buoyancy force, the governing equations are expressed as follows. Table 1 show the Property physic used in the present study

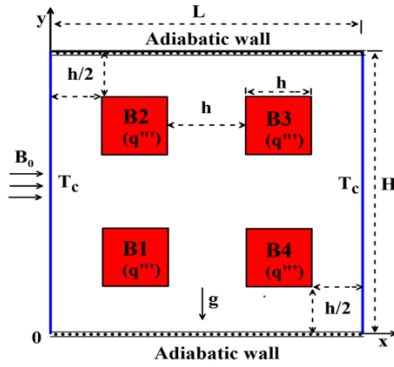


Figure 1. Detail of cases studied.

Table 1. Property physic used in the hybrid nanofluid

Property physic	H ₂ O	Cu	Al ₂ O ₃
ρ	997	8933	3970
Cp	4179	385	765
K	0,613	401	40
μ	9,09 10 ⁻⁰⁴	/	/
β	2,10 10 ⁻⁰⁴	1,67 10 ⁻⁰⁵	8,50 10 ⁻⁰⁶
σ	0,0000055	59600000	35000000
α	1,4713 10 ⁻⁰⁷	/	/

Density: $\rho_{hnf} = (1 - \phi_{hnf})\rho_f + (\phi_{Al_2O_3}\rho_{Al_2O_3} + \phi_{Cu}\rho_{Cu})$

where $\phi_{hnf} = \phi_{Al_2O_3} + \phi_{Cu}$

Heat capacity :

$(\rho c_p)_{hnf} = (1 - \phi_{hnf})(\rho c_p)_f + \phi_{Al_2O_3}(\rho c_p)_{Al_2O_3} + \phi_{Cu}(\rho c_p)_{Cu}$

Dynamic viscosity : $\mu_{hnf} = \mu_f (1 - \phi_{hnf})^{-2.5}$

Thermal expansion :

$(\rho\beta)_{hnf} = (1 - \phi_{hnf})(\rho\beta)_f + \phi_{Al_2O_3}(\rho\beta)_{Al_2O_3} + \phi_{Cu}(\rho\beta)_{Cu}$

Thermal conductivity :

$$\frac{k_{hnf}}{k_f} = \frac{\left(\frac{\phi_{Al_2O_3} k_{Al_2O_3} + \phi_{Cu} k_{Cu}}{\phi_{hnf}} \right) + 2k_f + 2(\phi_{Al_2O_3} k_{Al_2O_3} + \phi_{Cu} k_{Cu}) - 2\phi_{hnf} k_f}{\left(\frac{\phi_{Al_2O_3} k_{Al_2O_3} + \phi_{Cu} k_{Cu}}{\phi_{hnf}} \right) + 2k_f - (\phi_{Al_2O_3} k_{Al_2O_3} + \phi_{Cu} k_{Cu}) + \phi_{hnf} k_f}$$

Electrical conductivity:

$$\frac{\sigma_{hnf}}{\sigma_f} = \frac{\left(\frac{\phi_{Al_2O_3} \sigma_{Al_2O_3} + \phi_{Cu} \sigma_{Cu}}{\phi_{hnf}} \right) + 2\sigma_f + 2(\phi_{Al_2O_3} \sigma_{Al_2O_3} + \phi_{Cu} \sigma_{Cu}) - 2\phi_{hnf} \sigma_f}{\left(\frac{\phi_{Al_2O_3} \sigma_{Al_2O_3} + \phi_{Cu} \sigma_{Cu}}{\phi_{hnf}} \right) + 2\sigma_f - (\phi_{Al_2O_3} \sigma_{Al_2O_3} + \phi_{Cu} \sigma_{Cu}) + \phi_{hnf} \sigma_f}$$

Continuity equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (1)

Momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{hnf}} \frac{\partial p}{\partial x} + \nu_{hnf} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$
 (2)

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{hnf}} \frac{\partial p}{\partial y} + \nu_{hnf} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\sigma_{hnf} B_0^2}{\rho_{hnf}} v + \frac{(\rho\beta)_{hnf}}{\rho_{hnf}} g (T - T_c)$$
 (3)

Energy equation: $u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{hnf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$ (4)

Energy equation in heat sources: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -\frac{q'''}{\lambda}$ (5)

The specified problem's boundary conditions are outlined as follows:

Vertical walls: $T = T_c$ $u = v = 0$ (6)

Horizontal walls: $\frac{\partial T}{\partial x} = 0$ $u = v = 0$ (7)

Heating elements surfaces:

$$\frac{k_s}{k_{hnf}} \left(\frac{\partial T}{\partial \eta} \right)_s = \left(\frac{\partial T}{\partial \eta} \right)_{hnf} \text{ and } u = v = 0$$
 (8)

The subsequent dimensionless variables are introduced to convert the provided problem into a dimensionless form.

$$X = \frac{x}{H} \quad Y = \frac{y}{H} \quad U = \frac{uH}{\nu_{hnf}} \quad V = \frac{vH}{\nu_{hnf}}$$

$$Q = \frac{H^2}{k_f} Q_0 \quad P = \frac{pH^2}{\mu_{hnf} \nu_{hnf}} \quad \theta = \frac{(T - T_c)}{q''' H^2} \lambda$$

$$q''' = -\lambda \frac{T - T_c}{H^2} \quad Ra = \frac{g \beta_f \Delta T H^3}{\nu_f \alpha_f} = \frac{g \beta_f q''' H^5}{\alpha_f \nu_f \lambda}$$

$$Pr = \frac{\nu_f}{\alpha_f} \quad Ha = B_0 H \sqrt{\frac{\sigma_f}{\mu_f}}$$

Upon substituting the aforementioned variables, the equations

(1)-(8) can be expressed in dimensionless form as :

Continuity equation: $\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$ (9)

$$\text{Momentum equation: } U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (10)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - Ha^2 \left(\frac{\mu_f}{\mu_{hnf}} \right) \left(\frac{\sigma_{hnf}}{\sigma_f} \right) V + \frac{Ra}{Pr} \left(\frac{\beta_{hnf}}{\beta_f} \right) \left(\frac{\nu_f^2}{\nu_{hnf}^2} \right) \theta \quad (11)$$

Energy equation:

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left(\frac{1}{Pr} \right) \left(\frac{\alpha_{hnf}}{\alpha_f} \right) \left(\frac{\nu_f}{\nu_{hnf}} \right) \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (12)$$

$$\text{Energy equation in heat sources: } \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = 1 \quad (13)$$

The Nusselt number is utilized to assess the heat transfer rate from the surface of the heating element, defined in local and average forms, respectively, as follows:

$$Nu_\ell = -\frac{k_{hnf}}{k_f} \frac{\partial \theta}{\partial \eta} \quad \text{and} \quad Nu = \frac{1}{\ell} \int_0^\ell \left(-\frac{k_{hnf}}{k_f} \frac{\partial \theta}{\partial \eta} \right) d\zeta \quad (14)$$

3. NUMERICAL PROCEDURE

The system of dimensionless equations (9) - (13) governs the behavior of the fluid in the cavity. To solve the formulated problem, a numerical approach based on the finite volume method has been adopted. The domain is subdivided into uniform control volumes as illustrated in Figure 1. A 2D Cartesian coordinate system for an incompressible, stationary fluid with boundary conditions was assumed and implemented.

The resolution process was performed using the SIMPLE algorithm to solve the Navier-Stokes equations, this solver allows for the iterative correction of the coupling terms between velocity and pressure, ensuring numerical stability and the system convergence. A second-order Upwind scheme was used to discretize the convection and diffusion terms. A strict convergence criterion was set at 10^{-6} to ensure the accuracy of the numerical results.

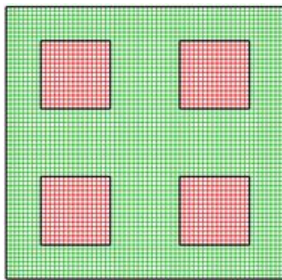


Figure 2. Detail of the computational grid within the enclosure.

To determine the mesh stability, the grid size has been uniformly varied from 20×20 to 200×200 for three different configurations represented in Figure 3. The average Nusselt number was calculated as a function of the grid size around the four heat-generating elements for each discretization. The best mesh configuration for all cases is 140×140 , where this value is optimal for all configurations.

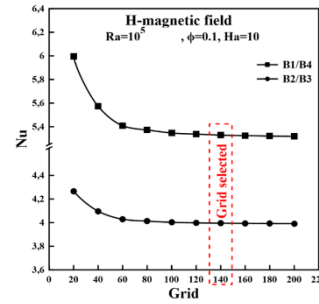
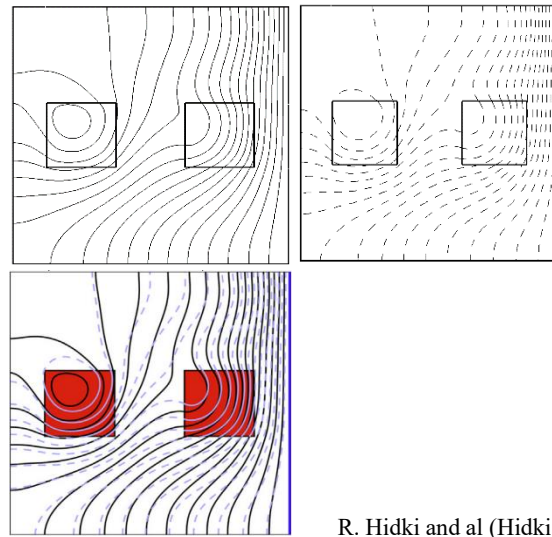


Figure 3. Effect of mesh size on the average Nusselt number along the different surfaces of the heating elements in the presence of a magnetic field.



P-Study

R. Hidki and al (Hidki et al., 2023)

Figure 4. Comparison between present work and the reference R. Hidki and al for the case of $Ra=10^5$

In order to validate the proposed numerical and physical models. Figure 4 present similar conditions between the Present study and the comparison literature the obtained results in this study provide a good evidence and coherence

4. RESULTS AND DISCUSSIONS

The behavior of natural convection was studied in a cavity filled with a hybrid nanofluid with a fraction ranging from 0 to 0.1 and Rayleigh range from 10^3 to 10^7 , in the presence of a magnetic field varying between 10, 25, and 50, applied horizontally or vertically. the square blocks generate heat with an intensity defined by (q''') . The obtained results are illustrated in this section as follows.

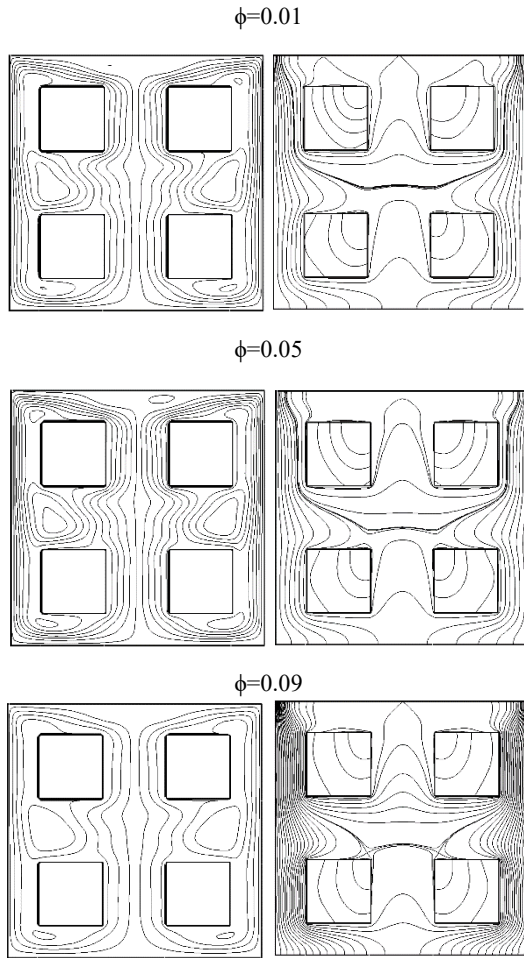


Figure 5. Effect of the volume fraction on the streamlines (on the right) and the isotherms (on the left) as a function of the $Ra=10^6$ with $Ha_x=50$

Figure 5 shows the evolution of streamlines (left) and isotherms (right) for different values of volume fraction. in the case of a number of $Ra=10^6$ and $Ha=50$, convection is the dominant mode, illustrated by the circles in the four blocks. the upper blocks have a higher temperature than the lower ones, due to the concentration of heat. the re-circulation zones show that the velocity is high in some regions of the cavity. the increase in the volume fraction improves heat transfer. the lower block has good re-cooling compared with the upper block. the heat is positioned significantly in the upper concentric part of the block, which shows that re-cooling is taking effect.

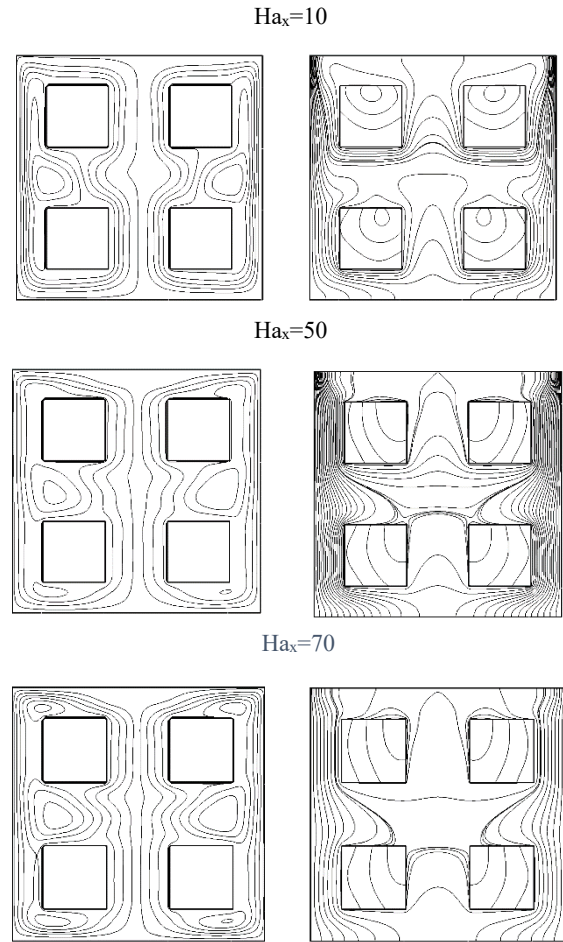


Figure 6. Effect of the Horizontal magnetic field on the streamlines (on the right) and the isotherms (on the left) as a function of the Rayleigh number with $Ra=10^6$ and $\phi=0.1$

Figure 6 shows the evolution of the streamlines (left) and isotherms (right) for different values of the Hartman number. increasing the Ha number decreasing the flow intensity, which is due to viscous forces dominating over magnetic ones. heat increases with increasing Ha in the cavity, which is caused by the deminimization of the flow intensity.

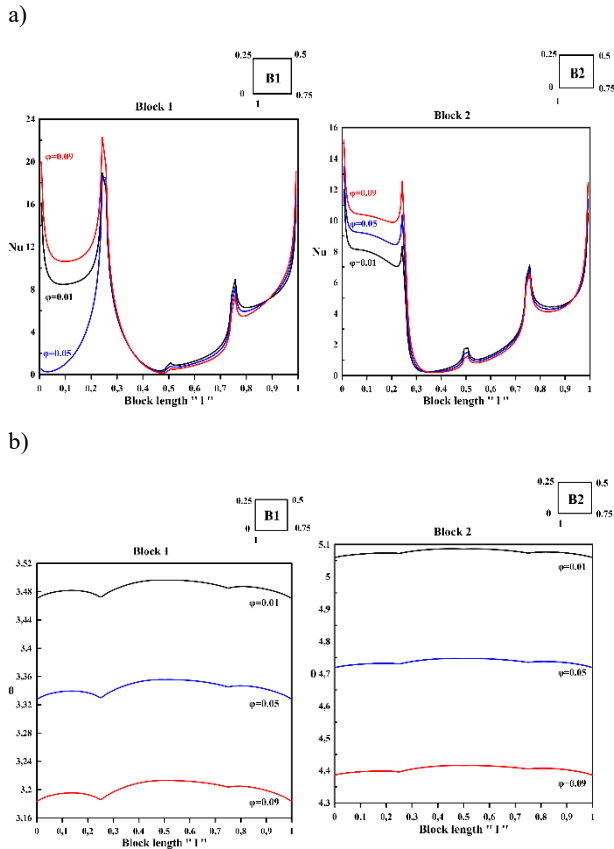


Figure 7. Variation of a) the local Nusselt number, b) temperature profile, for Different volume fraction with $Ra=10^6$ and $Ha=50$

Figure 7 shows the variation of the nusselt number and the temperature profile for block 1 and 2, at different values of the volume fraction. increasing the volume fraction increases the heat transfer and decreases the temperature of the blocks. the part parallel to the faroid roof has a better heat transfer than the other roofs. block 2 has a higher temperature and a lower heat transfer than block 1.

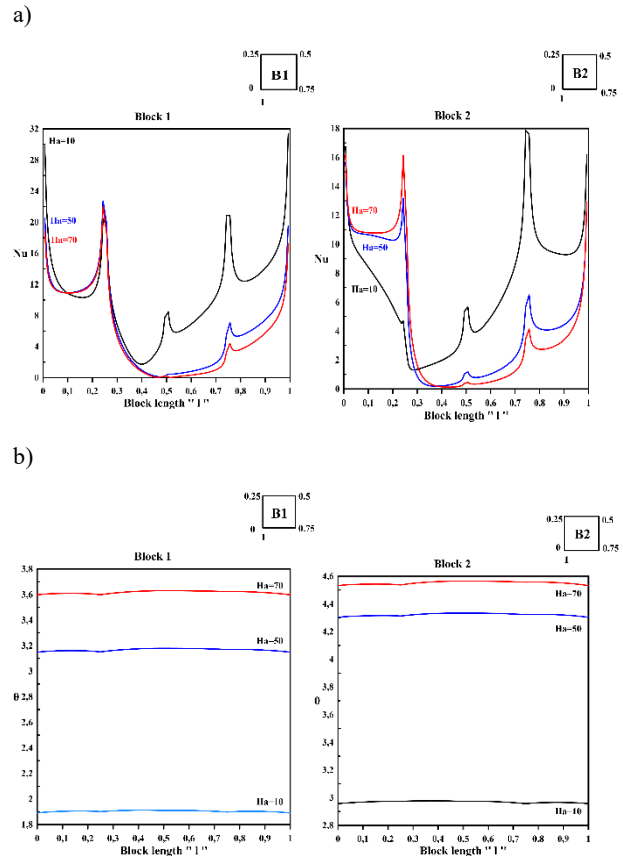


Figure 8. Variation of a) the local Nusselt number, b) temperature profile, for Different Hartmann numbers with $Ra=10^6$ and $\phi=0.01$

Figure 8 shows the variation in nusselt number and temperature profile for blocks 1 and 2, at different Hartmann number values. Increasing the Ha number deminimizes the heat transfer created by viscous forces. increasing the Ha number creates an increase in the block temperature, which is linked to the fluidity of flow in the cavity.

5. CONCLUSION

In this paper, a numerical study is conducted on natural convection of a hybrid nanofluid in a square cavity containing four heat-generating elements, in the presence of a horizontal magnetic field, using the finite volume method. The effects of the Rayleigh number, Hartmann number, nanoparticle volume fraction, local and average Nusselt number have been analyzed. The main results are summarized as follows

- Increasing the Rayleigh number and nanoparticle volume fraction enhances fluid flow and heat transfer within the cavity while reducing heat accumulation in the heat-generating blocks.
- An increase in the Hartmann number and internal heat generation reduces fluid flow and heat transfer within the cavity.

- The application of a horizontal magnetic field has a blocking effect on fluid flow and heat transfer.
- Regarding buoyancy force, heat transfer in block 1 is more significant than in block 2. Additionally, the sides of the square block parallel to the cold wall and the central recirculation zone exhibit the lowest heating.
- Thermal gradients are more pronounced on the upper sides of the cold wall, attributed to the concentration of heat at the top of the cavity.

REFERENCE

- Akhter, R., Mokaddes, M., Billah, M., & Uddin, N. (2023). Results in Engineering Hybrid-nanofluid mixed convection in square cavity subjected to oriented magnetic field and multiple rotating rough cylinders. *Results in Engineering*, 18(February), 101100. <https://doi.org/10.1016/j.rineng.2023.101100>
- Ali, O., Reza, M., Goodarzi, M., Sher, N., Zarringhalam, M., Ahmadi, G., Shabani, S., & Dahari, M. (2016). A modified two-phase mixture model of nanofluid flow and heat transfer in a 3-D curved microtube. *Advanced Powder Technology*, August, 1–11. <https://doi.org/10.1016/j.apt.2016.08.002>
- Chamkha, A. J., Dogonchi, A. S., & Nabwey, H. A. (2021). MHD mixed convection of localized heat source / sink in an Al₂O₃-Cu / water hybrid nanofluid in L-shaped cavity. *Alexandria Engineering Journal*, 60(3), 2947–2962. <https://doi.org/10.1016/j.aej.2021.01.031>
- Choi, S. U. S. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *American Society of Mechanical Engineers, Fluids Engineering Division (Publication) FED*, 231, 99–105.
- Ganesh, N. V., Javed, S., Al-mdallal, Q. M., Kalaivanan, R., & Chamkha, A. J. (2020). *Heliyon Numerical study of heat generating γ Al₂O₃ – H₂O nano fluid inside a square cavity with multiple obstacles of different shapes*. 6(August). <https://doi.org/10.1016/j.heliyon.2020.e05752>
- Hidki, R., Moutaouakil, L. El, Boukendil, M., Charqui, Z., Zrikem, Z., & Abdelbaki, A. (2023). Materials Today : Proceedings Impact of Cu , Al₂O₃ -water hybrid nanofluid on natural convection inside a square cavity with two heat-generating bodies. *Materials Today: Proceedings*, 72, 3749–3756. <https://doi.org/10.1016/j.matpr.2022.09.292>
- Hossain, R., Azad, A. K., Jahid Hasan, M., & Rahman, M. M. (2022). Thermophysical properties of Kerosene oil-based CNT nanofluid on unsteady mixed convection with MHD and radiative heat flux. *Engineering Science and Technology, an International Journal*, 35, 101095. <https://doi.org/10.1016/j.jestch.2022.101095>
- Ismael, M. A., Mansour, M. A., Chamkha, A. J., & Rashad, A. M. (2016). Mixed convection in a nanofluid filled-cavity with partial slip subjected to constant heat flux and inclined magnetic field. *Journal of Magnetism and Magnetic Materials*, 416, 25–36. <https://doi.org/10.1016/j.jmmm.2016.05.006>
- Kavusi, H., & Toghraie, D. (2017). A comprehensive study of the performance of a heat pipe by using of various nanofluids. *Advanced Powder Technology*, October, 1–11. <https://doi.org/10.1016/j.apt.2017.09.022>
- Khodabandeh, E., Reza, M., Akbari, S., Ali, O., & Alrashed, A. A. A. (2018). Application of nano fluid to improve the thermal performance of horizontal spiral coil utilized in solar ponds : Geometric study. *Renewable Energy*, 122, 1–16. <https://doi.org/10.1016/j.renene.2018.01.023>
- Mansour, M. A., Rashad, A. M., Gorla, R. S. R., & Siddiqua, S. (2016). Inclined Magneto-Hydrodynamic Mixed Convection in Lid-Driven Cavity Filled Within Nanofluids with Partial Slip and Internal Heat Generation. *Journal of Nanofluids*, 5(4), 634–651. <https://doi.org/10.1166/jon.2016.1246>
- Marandi, O. F., Ameri, M., & Adelshahian, B. (2018). The experimental investigation of a hybrid photovoltaic-thermoelectric power generator solar cavity-receiver. *Solar Energy*, 161(November 2017), 38–46. <https://doi.org/10.1016/j.solener.2017.12.039>
- Selimefendigil, F., & Öztop, H. F. (2019). MHD mixed convection of nanofluid in a flexible walled inclined lid-driven L-shaped cavity under the effect of internal heat generation. *Physica A*, 534, 122144. <https://doi.org/10.1016/j.physa.2019.122144>
- Tasnim, S., Mitra, A., Saha, H., Islam, Q., & Saha, S. (2023). Results in Engineering MHD conjugate natural convection and entropy generation of a nanofluid filled square enclosure with multiple heat-generating elements in the presence of Joule heating. *Results in Engineering*, 17(February), 100993. <https://doi.org/10.1016/j.rineng.2023.100993>