DEGRADATION MODELS FOR RELIABILITY AND REMAINING USEFUL LIFE ESTIMATION: APPLICATION ON CRACK PROPAGATION

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Abstract: This article delves into the important role of degradation models and measurement methodologies in estimating equipment reliability and predicting its Remaining Useful Life (RUL). Such predictions are important in alleviating costly unplanned maintenance while increasing machine reliability, availability and safety. If an equipment is subjected to a degradation process on which a critical threshold may be fixed, a degradation model may be fitted and used to predict the failure times. Thus, it is possible to track this degradation to retrieve the health state of the system from the set of degradation model points. This paper presents a stochastic degradation model and discusses the effects of a dispersion of degradation model parameters on the degradation. It presents a RUL model based on retrieving reliability law from a degradation model and compare it with different RUL models which lead to RUL evaluating of this model.

Keyword: degradation, physical models, crack propagation, reliability, RUL

1. INTRODUCTION

Industrial maintenance plays a great role in ensuring the operational, reliability and safety of industrial equipment and systems. To develop a maintenance strategy, degradation models can serve as its foundation. Knowing how longer a machine can function is key for planning maintenance activities. This involves analyzing indicators to establish laws that describe how this process changes over time. Usually, this approach allows to predict how longer equipment will survive and develop a maintenance plan to prevent breakdowns.

Degradation modeling can generally be categorized into three main approaches: physical models, data-driven models, and hybrid models. Physical models aim to capture the mechanisms and physical processes that drive equipment failure. In contrast, data-driven models capture the degradation trends directly from available data without requiring detailed knowledge of the specific failure modes. Hybrid models combine both approaches to not only integrate the strengths of both data-driven and model-based approaches but also alleviate their limitations.

Modeling degradation using physical models continues to attract considerable interest, as it directly incorporates the different parameters influencing the degradation process. Several studies have been conducted using physical models for the RUL assessment. Swanson et al. proposed a Kalman filter method to model the crack growth in a tensioned steel band, where RUL was derived using the predicted model frequency state when it reached a predefined failure threshold (Swanson et al., 2000). Si et al. developed a recursive Kalman filter algorithm for RUL estimation by integrating the property of Wiener process and the updating capability of the Kalman filter (Si et al., 2013). The assumption of using a Kalman filter is that the process must be linear. However, when dealing with a non-linear system that is not constrained by the linear Gaussian noise assumption, which is more prevalent in industrial systems, particle filters are typically the dominant choice.

Miao et al. proposes a particle filtering and smoothing framework for prognostics and health management (PHM) (Miao et al., 2014). The framework uses particle filters to handle nonlinear and non-Gaussian problems in degradation modeling and RUL prediction, the degradation process is represented using a state-space model while the RUL is predicted based on the estimated state trajectory and a predefined failure threshold. A smoothing method is also integrated to improve the estimation accuracy. Saha et al. applied particle filter to estimate the coefficients of an exponential growth model for electrolyte resistance and charge transfer resistance and RUL was calculated as the interval between the current cycle and the end-of-life with the developed degradation model (Saha et al., 2007). Bolander et al. developed a spall propagation model for aircraft engine bearings, and used a Particle filter-based approach to predict the RUL (Bolander et al., 2009). Liao used the Pairs–Erdogan model to describe the fault growth of bearings, and predicted the RUL using a continuous Bayesian updating approach (Liao, 2014). Zhao et al. presented a prognostics method for RUL prediction for gears (Zhao et al., 2013). The crack growth process was simulated to yield normally distributed crack lengths, which were used within a Bayesian framework to update the parameters of the degradation model (e.g., Paris' law). The degradation model was initially fed by the results of a stress analysis from a gear dynamic model or finite element model. The distributions of the uncertainty factors were updated via Bayesian inference using the condition monitoring data (simulated crack lengths), and an estimation of the RUL based on the degradation model was provided.

Physical degradation models can be generally expressed in different fundamental forms like power and exponential law. These forms suggest a potential mapping relationship between mathematical models and the degradation mechanisms caused by different parameters. It is established that temperature-related degradation often follows an exponential relationship, while mechanical stresses typically can be expressed with a power law form. Gebraeel et al. used the exponential form to describe the degradation propagation of gears and used the Bayesian method to combine the historical data with the current detection data to predict the RUL life of gears in real time (Gebraeel et al., 2005). In addition, Takeda et al. used the power law form to study the degradation process of electric device and RUL was derived using the predicted degradation model coupled with a bayesian method to combine the historical data (Takeda et al.,1983).

Based on the state of the art, it is crucial to propose a reliability modeling approach that incorporates a physical degradation law. This approach is essential for planning maintenance strategies and estimating the RUL. By coupling the physical modeling of failures with the First Hitting Time Threshlod ($FHTT$) method, it is possible to develop a theoretical reliability degradation model and perform RUL estimation based on this theoretical model.

$FHTT$ refers to the duration it takes for a system's performance to reach a certain predefined threshold, beyond which it is considered to have failed or degraded to an unacceptable level. This threshold could represent various factors depending on the context, such as a critical level of wear in mechanical systems. $FHTT$ models consist of stochastic degradation function $Z(t)$ that represents the evolution of a degradation process over time. Beyond a predefined threshold Z_c ,the degradation is considered critical and requires intervention, which can be summarized by:

$$T_c = inf\{t|Z(t) > z_c\} \quad (1)$$

T_c : represents the random variable denoting the first passage time.

$inf\{t|Z(t) > z_c\}$: denotes the infimum (greatest lower bound) over all times t such that the stochastic process $Z(t)$ exceeds the threshold z_c. In other words, it represents the smallest time at which the process $Z(t)$crosses or exceeds z_c. This definition implies that it is possible to observe an unknown stochastic process evolution once it exceeds the specified threshold. For each simulation i, the observed domain is therefore a random interval $[0, T_{ci}]$ since at time T_{ci}, the failure is observed ($Z(T_{ci}) = Z_{ci}$). The choice of a stochastic degradation model depends on the observed physical process. From (1), the reliability is given by:

$$R(t) = P(Z^{-1}(z_c) > t) \qquad (2)$$

and the Mean Residual Life (MRL) is defined by:

$$MRL(t) = \frac{\int_t^\infty R(y)dy}{R(t)} \quad (3)$$

This study aims to employ physical modeling to retrieve failure data from a simulated model with performing a dispersion on the law parameters. The objective is to determine the distribution of the simulated data with statistical test. Additionally, performing a degradation model analysis to enhance reliability assessment and predict the RUL. This analysis leverages $FHTT$ method to refine the reliability prediction. By comparing the theoretical reliability model with the best adjusted model derived from simulated data, aiming to highlight the improvements in reliability assessment and RUL prediction achieved through these methodologies.

2.Case study

2.1 application on crack propagation

This study concerns the growth of a crack in a mechanical equipment. The used degradation model is the simplified Paris relationship (Paris & Erdogan, 1963), which links the evolution of the crack size a with the number of cycles N by :

$$\frac{da}{dN} = C\Delta k^m \qquad (4)$$

With, C and m are constants that depend on material properties, and Δk is the range of the stress intensity factor defined by:

$$\Delta k = (\sigma_{max} - \sigma_{min})\sqrt{\pi a} \quad (5)$$

Where, σ_{max} , σ_{min} are the tensile stress and compressive stress with σ_{min} fixed to 0 MPa as a compressive stress tends to close the crack, so the relation (5) would be:

$$\Delta k = \sigma_{max}\sqrt{\pi a} \quad (6)$$

The parameters C and m are determined experimentally based on fatigue tests. The values obtained for these parameters thus subject to uncertainty. To represent this uncertainty, C is distributed according to a Weibull distribution.

The case studied is derived from (Dowling et al., 2013), where a specimen is made of aluminum alloy $7075 - T6$ (W = 152.4 mm, H = 445 mm, T = 2.29 mm,), as it is illustrated in figure 1. The corresponding stresses are $\sigma_{min} = 0$ MPa, $\sigma_{max} = 69$ MPa and the initial crack length is $a_0 = 5$ mm and the critical crack length is $a_c = 45$ mm. The value of m parameters is 3.25

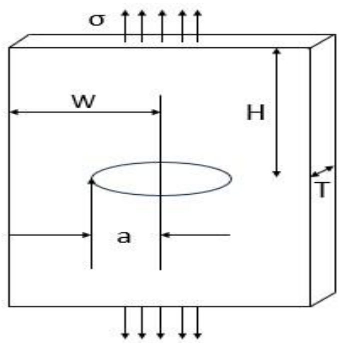

Figure 1: tested specimen

2.2 Parameter dispersion

The parameter C is suggested to be distributed according to a Weibull distribution with the above parameters (a_0, a_c, σ, m, $C \approx$ weibull (β_c, η_c)) with β_c and η_c are the shape and scale parameters of the Weibull distribution law respectively. The scale parameter is set equal to the nominal value of C, which is η_c=9.1*$10^{-11} \frac{m}{cycles}(MPa\sqrt{m})^{-m}$, while the shape parameter is fixed to 3 .
the crack growth curves obtained for 1000 simulations are shown in figure 2.

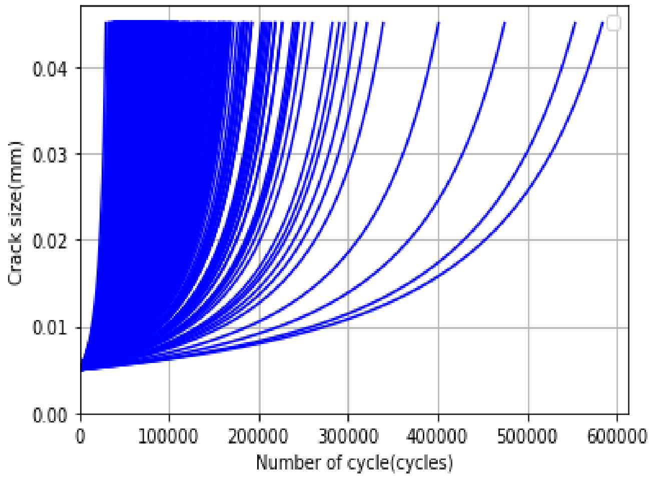

Figure 2. Evolution of crack size over 1000 simulations.

To analyze failure times different statistical distribution laws are used, such as: log-normal and Weibull distributions (fitted with both the Product Limit (PL) and maximum likelihood estimation method (MLE)). The Anderson-Darling (AD) score was chosen for statistical tests due to its effectiveness. The lowest value, closest to 0, indicates the best statistical distribution that can be fitted to the simulated data, with AD:

$$AD = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)[\ln(F(T(i))) + \ln(1-F(T(n\ 1-i)))] \quad (7)$$

Where:

AD : the Anderson-Darling test statistic and its calculated value based on the discrepancies between the observed simulated data and the hypothesized distribution:

n : is the sample size.
x(i) : is the $i-th$ is the ordered observation.
F(T) : is the cumulative distribution function (CDF) of the hypothesized distribution.
Figure 3 shows the fitted $F(T)$ for the simulated data using different statistical distributions based on 1000 simulations.

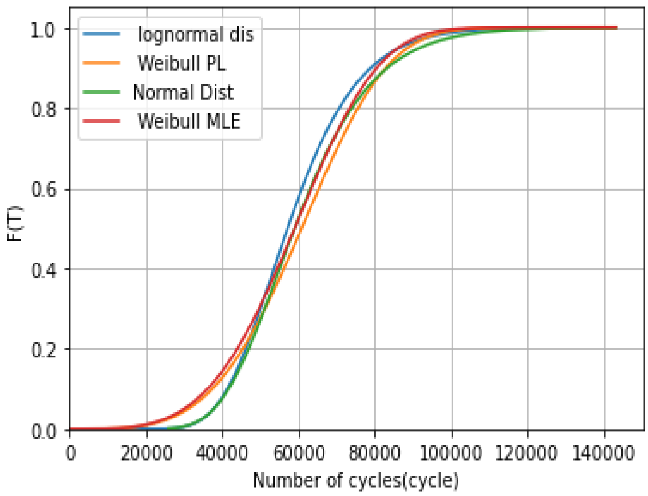

Figure 3: F(T) of different statistical distributions fitted for $n = 1000$ simulations.

Table I: Correlation results and AD score of different statistical distributions

	Weibull - MLE	Log-normal	Normal	Weibull-pl
AD	1.62	1.12	2.15	0.38

In comparison between the AD score results, it can be seen that the most correlated distribution to the simulated data is the $Weibull - PL$ compared to the log-normal and normal distributions. Hence, the $Weibull\ PL$ distribution law of the simulated results is represented by:

$$F(t) = \frac{\beta}{\eta}(\frac{t-\gamma}{\eta})^{\beta-1}\exp(-(\frac{t-\gamma}{\eta})^{\beta}) \quad (8)$$

β : the shape factor.
η: the scale parameter.
γ : time offset parameter.

The estimated parameters of the fitted Weibull distributions law are ($\beta = 3.08, \eta = 64900, \gamma = 1/\eta$).

Based on the results obtained from various statistical adjustments for reliability analysis as outlined above and according to (Sun et al., 2013), it is possible to calculate the MRL, which is depicted in Figure 4 for $Weibull\ PL$ model

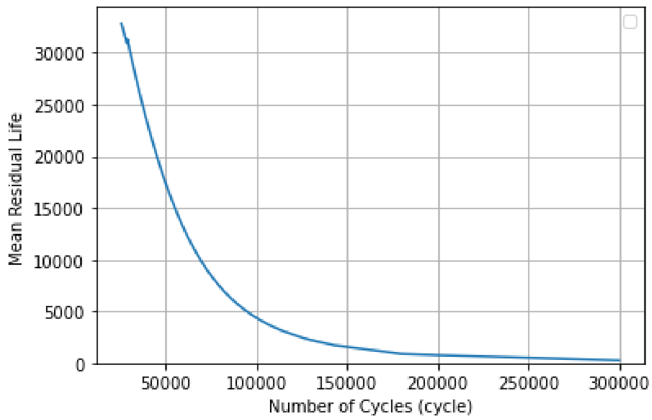

Figure 4: MRL evolution of Weibull Pl model

3.Degradation model analysis

This section delves into the integration of $FHTT$ on physical models, particularly focusing on the Paris law. The Paris law accounts for the evolution of crack growth according with the number of cycles, as represented by (4):

$$\frac{da}{dN} = C\sigma^m (\pi a)^{m/2} \quad (9)$$

$$\frac{1}{dN} = \frac{C\sigma^m (\pi a)^{m/2}}{da} \quad (10)$$

$$N_c = \frac{1}{C\sigma^m \pi^{m/2}} \int_{a_0}^{a_c} a^{-m/2}\, da \quad (11)$$

$$N_c = \frac{a_c^{\left(-\frac{m}{2}\right)+1} - a_0^{\left(-\frac{m}{2}\right)+1}}{\left(-\left(\frac{m}{2}\right)+1\right)\left(C\sigma^m \pi^{\frac{m}{2}}\right)} \quad (12)$$

Thus, the number of cycles to critical failure (N_c) may be calculated by the above equation if the parameters are known.

$a_0 = 5$ mm
$a_c = 45$ mm
$\sigma = 69.91$ MPa
$m = 3.25$
$C = 9.1 * 10^{-11} \frac{m}{cycles} (MPa\sqrt{m})^{-m}$

The obtained critical number of cycles is $N_c = 56723$ cycles.

The reliability function may be expressed by:

$$R(N) = P\,(N_c > N) \quad (13)$$

$$R(N) = P\left(\frac{a_c^{\left(-\frac{m}{2}\right)+1} - a_0^{\left(-\frac{m}{2}\right)+1}}{\left(-\left(\frac{m}{2}\right)+1\right)\left(C\,\sigma^m \pi^{\frac{m}{2}}\right)} > N\right) \quad (14)$$

$$R(N) = P\left(C < \frac{a_c^{\left(-\frac{m}{2}\right)+1} - a_0^{\left(-\frac{m}{2}\right)+1}}{\left(-\left(\frac{m}{2}\right)+1\right)\left(N\sigma^m \pi^{\frac{m}{2}}\right)}\right) \quad (15)$$

than the failure function would be:

$$F(N) = 1 - R(N) \quad (16)$$

knowing that C is a stochastic value following a Weibull distribution law $W\,(\beta_c, \eta_c)$:

$$P(c < X) = 1 - \exp\left(-\left(\frac{X}{\eta_c}\right)^{\beta_c}\right) \quad (17)$$

So, the reliability function of the degradation model would be:

$$R(N) = 1 - \exp\left(-\left(\frac{a_c^{\left(-\frac{m}{2}\right)+1} - a_0^{\left(-\frac{m}{2}\right)+1}}{\left(-\left(\frac{m}{2}\right)+1\right)\left(N\eta_c\, \sigma^m \pi^{\frac{m}{2}}\right)}\right)^{\beta_c}\right) \quad (18)$$

Figure 5 shows the results of the obtained F(T) from (16) and (18) of the theoretical degradation model.

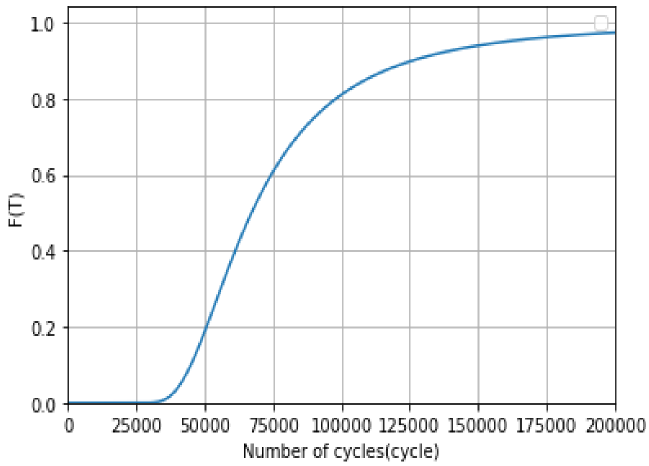

Figure 5: F(t) of theoretical degradation model

Figure 6 shows the evolution of the MRL obtained for the theoretical degradation model.

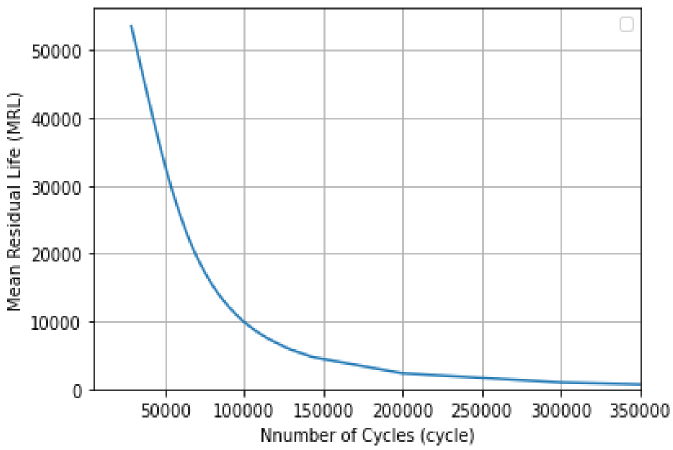

Figure 6: Evolution of MRL for the theoretical degradation model

The reliability obtained by the predicted degradation model helps in predicting the MRL for a given damage index DI, defined by:

$$DI\,(N_i) = \frac{Z(N_i) - Z(0)}{Z(N_c) - Z(0)} \quad (19)$$

So, in the present study, the degradation index expression would be:

$$DI(N_i) = \frac{a(N_i) - a(0)}{a(N_c) - a(0)} \qquad (20)$$

$$0.5 = \frac{a(N_{0.5}) - 0.005}{0.045 - 0.005} \qquad (21)$$

the mean crack length $a(N_{0.5})$ for $DI = 0.5$ is 25mm so the mean number of $N_{0.5}$ may be calculated with (12), Then the equivalent number of cycles is $N_{0.5} = 52721$ cycles and according to (3) the mean residual life will be:

$$MRL(N_{0.5}) = \left(\frac{\int_{52721}^{\infty} R(N)\, dN}{R(52721)}\right)$$

So, the MRL for an observed $DI = 0.5$ before the crack reaches the threshold worth 29711 cycles.

4.Models comparison

Figure 7 provides a MRL evolution of the $Weibull - Pl$ model and the theoretical degradation model.

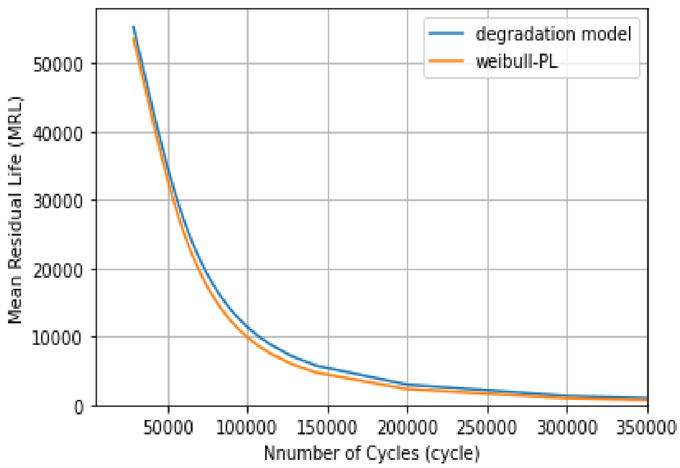

Figure 7: Comparison between the MRL evolution between the Weibull PL and the theoretical degradation model.

The correlation coefficient between the MRL of the theoretical degradation model and the MRL of the $Weibull - PL$ model is 0.95. This high correlation proves that the $Weibull - PL$ model offers good results in predicting the MRL for the simulated data points. This demonstrates the effectiveness of the $Weibull - PL$ model in modeling degradation and predicting maintenance needs in industrial settings.

5. CONCLUSION

This study employs a physical modeling approach using the Paris-Erdogan model for crack growth propagation. Parameter dispersion performed to observe its effect on degradation evolution and predict failure times. The distribution of simulated data was determined using AD test. The Weibull product limit model proved to be the best fit for the simulated data. A degradation model analysis was conducted to enhance reliability assessment and predict the remaining useful life (RUL) of the asset, supported by the ($FHTT$) method. Comparing the best-fitted $Weibull - PL$ model to the theoretical degradation model revealed that it offers a high correlation. This proves that the $Weibull - PL$ model provides good results in predicting the MRL for the simulated data points. Future work aims to develop a hybrid degradation model that combines the strengths and mitigates the weaknesses of both data-driven and physics-based models, optimizing RUL estimations and informing maintenance policy.

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